Historical Dimensions of Mathematics

- It gives deepere understanding of mathematics itself.
- It is essential to understand the history of how maths evolved in order to understand the evolution of physics, engineering, computer science etc.
- Helps to know the lives and contributions of mathematics and to be greatful towards their contributions.
- Enhance motivation towards learning the subject and develop postive attitude towards mathematics.
- It helps students to acquire a balance between "rigore" and "imaginations".
- The history helps to explain the role of mathematics in society, shows the development of mathematics as a human activity.

- Helps to create a lively classroom environment and to increase student interest towards learning.
- Past obstacles in the development of mathematics can help to explain what today's students find difficult.
- It gives a guide towards teaching.
- It helps to develop meaningful relationship between abstract knowledge and real-world application in a real environment.

Scientific Dimension of Mathematics.

1. Mathematics is the science of quantity and space.
2. Mathematics is the science of calculation.
3. Mathematics is the method of the progress of various subjects.
4. It is the abstract form of science.
5. It is a systematized, organised and exact branch of science.
6. It is an important means of generalization.
7. It is the means of drawing conclusion and judgement.
8. It is the science of logical reasoning.

Moves of Teaching Models

In context of teaching "moves" refers to deliberate strategies, techniques and actions that one can use to facilitate learners in concept formations.

(i) Stating necessary Condition:-

A condition 'A' is said to be necessary for a condition B, if (and only if) the falsity of A guarantees the falsity of B.

Eg:- "Air is necessary of human beings".

(ii) Sufficient Conditions:-

A condition 'A' is said to be sufficient for a condition B; if the truth of A guarantees the truth of B.

Eg:- "Four equal sides with 90° each angle is a square".

(iii) Giving Example with reason and without reason:-

A specific instance that illustrate a concept, definition or statement. It shows that a particular scenario or case satisfies the condition or properties described.

with reason:-

"The number 4 is an even number because it can be divided by 2 without leaving a remainder".

This example provides a reason (divisible by 2) to justify why 4 is an even number.

Without reason:-

"The number 4 is an even number": This example simply states that 4 is an even number without providing a justification or reason.

(iii) Non-Example with and without reason:-

It is an instance that doesnot fit the concept, definations or statement. It shows that a particular case doesnot satisfy the condition or properties described. It can helps clarify boundries and limitions.

With reason:-

"The number 3 is not an even number because it leaves a reminder of 1 when divided by 2.

This non-example provides a reason (leaves a remainder) to explain why 3 does not fit the concept of an even number.

without reason!-

"The number 3 is not an even number", This non-example simply states that 3 doesn't fit the concepts of an even-number without providing justification or reason.

(iv) Providing Counter Example!-

It is a specific instance that disproves a statement, conjecture or hypothesis. It shows that a particular case contradicts they claims, demonstrating that the statement is false or incomplete. It is crucial in mathematics and logic to test and define theories

The Statement "If n^2 is odd then n is odd" is a counter example as "If we consider n is not odd then n^2 is not odd.

i.e Let $n = 2$ (not odd)

$n^2 = 2^2 = 4$ (not odd)

Therefore, the Statement "If 4^2 is odd then n is odd," is false and our counter example proves it.

Thus from the above, we can conclude that for an effective teaching-learning process, one can use different range of moves to create a dynamic and supportive learning environment.

Example of Inductive Method.

- Find the S.I on Rs 500 at the rate of 5% per annum for a period of 3 years.

Interest on Rs 100 for 1 yr = Rs 5

" " " 100 for 3 yr = Rs (5×3)

" " " 1 " " = $\frac{5\times3}{100}$

" " " 500 " " = $\frac{5\times3}{100}\times500$

= Rs 75.

Example of Deductive method.

- Solve $(2a+3b)^2$

We know $\Rightarrow (a+b)^2 = a^2+2ab+b^2$

$\therefore (2a+3b)^2 = (2a)^2 + 2\cdot(2a)\cdot(3b) + (3b)^2$

$= 4a^2 + 12ab + 9b^2$

Direct proof

* Prove that if n is an odd integer then n^2 is also an odd integer.

Let, P be the statement that n is an odd integer.

and, Q be the statement that n^2 is an odd integer.

Assume, that n is an odd integer, then by definitions, $n = 2k+1$ for some integer k. We will now use this to show that n^2 is also an odd integer.

$$n = (2k+1)$$

$$n^2 = (2k+1)^2$$

$$= (2k)^2 + 2 \cdot 2k \cdot 1 + 1$$

$$= 4k^2 + 4k + 1$$

$$= 2(2k^2 + 2k) + 1$$

Hence we have shown that n has the form of an odd integer since $2k^2+2k$ is an integer.

Therefore we have shown that $P \Rightarrow Q$ and so we have completed our proof.

Indirect Proof :-

* Proof by Contradiction :-

- Prove $\sqrt{2}$ is irrational number

Let, $\sqrt{2}$ be a rational number.

$\therefore \sqrt{2} = \frac{a}{b}$ ——— (1).

where, a and b are integers and a and b have no common factors and $b \neq 0$.

Sq. both sides of (1)

$\Rightarrow (\sqrt{2})^2 = \left(\frac{a}{b}\right)^2$

$\Rightarrow 2 = \frac{a^2}{b^2}$

$\Rightarrow a^2 = 2b^2$ ——— (11)

If a^2 is multiple of 2.

then a is multiple of 2.

Again,

Let, $a = 2c$ (c is integer)

$\Rightarrow a^2 = (2c)^2$

$\Rightarrow a^2 = 4c^2$ ——— (i)

$\Rightarrow 2b^2 = 4c^2$ (from (i))

$\Rightarrow b^2 = 2c^2$ ——— (ii)

$\therefore b^2$ is multiple of 2.

b is multiple of 2.

From (i) & (ii) we get that 2 is common factor.

——— This is a contradiction.

Thus $\sqrt{2}$ is irrational.

Disproof by Counter Example:

- If n^2 is odd, then n is odd.

Sol// If n is not odd, then n^2 is not odd.

$\therefore n = 2m$; $m \in \mathbb{I}$.

Again, $n^2 = (2m)^2 = 4m^2$

$= 2(2m^2)$

which is not odd.

Hence, the given statement is false.

Proof by Cases.

Prove by induction; $\forall n$, $1+2+3+\cdots+n = \frac{n(n+1)}{2}$

To prove;

$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

Case I $\Rightarrow$ for $n=1$.

$$1 = \frac{1(1+1)}{2} = \frac{1\times 2}{2} = 1.$$

Case II $\Rightarrow$ Let $n=k$.

$$1+2+3+\cdots+k = \frac{k(k+1)}{2} \quad ; \text{ for some } k. \qquad \text{—— (1)}$$

Case - III Let $n=k+1$.

$$1+2+3+\cdots+(k+1) = \frac{(k+1)(k+1+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}.$$

$\therefore$ L.H.S $\Rightarrow$

$$1+2+3+\cdots+(k+1)$$

$$= 1+2+3+\cdots+k+(k+1)$$

$$= \frac{k(k+1)}{2} + (k+1) \qquad [\text{from (1)}]$$

$$= \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$$= R.H.S \;//$$

Constructivism & Enactivism.

1. Constructivism maintains seperation of mental thought from the physical action, whereas, the two are combine in enactivism.

2. Knowledge according to constructism is an object that occupies space and must be build by the learner with facilitation from a teacher, whereas, in enactivism knowledge is present in all things and in all action and the teacher and students discover the knowledge together as it co-evolves with the action they have.

3. Constructivism is concerned only with cognitive knowledge and doesnot consider subconscious knowledge one the role of emotions in learning.

4. Making connections is an important part of Constructionism.

Gaining knowledge is a focus of constructivism and knowledge about interaction of environments is the focus of enactivism.